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Intensity-dependent scattering energies in high-intensity Møller scattering

J Bös†, W Brock†, H Mitter‡ and Th Schott†

† Institut für Theoretische Physik der Universität, D–7400 Tübingen, West Germany
‡ Institut für Theoretische Physik der Universität, A–8010 Graz, Austria

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Abstract. For Møller scattering in a monochromatic laser field an energy conservation law is valid which differs considerably from that for free Møller scattering. The consequences of this modified conservation law for the scattering energy are examined for strong circularly polarised laser fields. The laser field acts in two ways on the electrons, which experience both the collective phenomenon of an intensity-dependent energy shift, and can also absorb or emit laser quanta. The second effect produces a discrete energy spectrum. In most cases the intensity-dependent shift is covered by this discrete spectrum. For both effects approximate formulae and numerical results for the scattering energy of non-relativistic electrons are presented and the possibility of an experimental verification of these energy changes is discussed.

1. Introduction

In a previous paper on Møller scattering in a monochromatic circularly polarised laser field, resonances and intensity-dependent shifts of the off-resonance cross section were examined (Bös *et al* 1979). In calculating the amplitude of this scattering process an energy–momentum conservation law was found which differed considerably from free Møller scattering. (The notation is the same as in the previous paper; natural units are again employed):

$$\tilde{p}'_1+\tilde{p}'_2-\tilde{p}_1-\tilde{p}_2-rk=0. \tag{1.1}$$

$\tilde{p}'_1, \tilde{p}'_2$ are the effective 4-momenta of the scattered electrons, $\tilde{p}_1$ and $\tilde{p}_2$ those of the incoming electrons, k is the wavevector and r is the net number of laser quanta absorbed or emitted by both electrons. The effective momentum was defined by

$$\tilde{p}:=p+\frac{(ea)^2}{2p\,.\,k}k$$

where a is the amplitude of the laser field. The effective momentum fulfils a different mass shell relation:

$$\tilde{p}^2=m^2(1+\nu^2)=:m_*^2$$

where ν^2 is a dimensionless parameter characterising the intensity of the laser field:

$$\nu^2:=\left(\frac{ea}{mc^2}\right)^2.$$

Hence the laser field acts on the electrons in two ways:

(i) The moments are shifted from p to $\tilde{p}$, whereby the electrons obtain an effective mass m_*. This effect can be explained by classical electrodynamics; the shift depends only on the (classical) parameter ν^2 (Brown and Kibble 1964, Kibble 1965). For the quantum mechanical description of the effect it is important to recall that the electrons can emit and absorb a high number of laser quanta.

(ii) If the number of the emitted laser quanta differs from the number of absorbed quanta, a discrete spectrum of momenta will be found in addition to the ν^2 dependent shift.

The modified conservation law leads to a new condition for the scattering energy E_1':

$$\tilde{p}_2'^2 = (\tilde{p}_1 + \tilde{p}_2 - \tilde{p}_1' + rk)^2 = m_*^2. \tag{1.2}$$

When no laser field is applied E_1' is easily calculated from the quadratic equation

$$(p_1 + p_2 - p_1')^2 - m^2 = 0.$$

In the centre of mass system (CMS) one simply obtains $E_1 = E_2 = E_1' = E_2'$. Equation (1.2), however, leads to an equation with fourth powers in E_1' and cannot be simplified by introducing a CMS. In general neither $E = E_1'$ nor $E = E_2'$ nor $E_1' = E_2'$ will be valid. Nevertheless, we introduced a CMS for the incoming electrons in our previous paper, because the formulae turned out to be shorter than for the laboratory system. In this CMS the momenta are

$$p_1 = (E, \boldsymbol{p}), \qquad p_2 = (E, -\boldsymbol{p}), \qquad p_1' = (E_1', \boldsymbol{p}_1'), \qquad p_2' = (E_2', \boldsymbol{p}_2'),$$

and the angles are defined by

$$\theta := \sphericalangle(\boldsymbol{p}, \boldsymbol{p}_1'), \qquad \psi := \sphericalangle(\boldsymbol{p}, \boldsymbol{k}), \qquad \phi := \sphericalangle(\boldsymbol{p}_1', \boldsymbol{k}).$$

In the limit of a vanishing external field this CMS goes over into the normal CMS. It describes an experimental situation where the electrons come into the laser beam with equal and opposite momenta and interact within the laser field.

The scattering angle must satisfy the condition

$$|\psi - \phi| \leq \theta \leq \min(\psi + \phi, 2\pi - \phi - \psi).$$

In this system equation (1.2) reads

$$2E^2 - 2EE_1' + r\omega(2E - E_1' + |\boldsymbol{p}_1'| \cos\phi) + \nu^2 m^2 E\left(\frac{2E - E_1' + |\boldsymbol{p}_1'|\cos\phi}{E^2 - \boldsymbol{p}^2\cos^2\psi} - \frac{1}{E_1' - |\boldsymbol{p}_1'|\cos\phi}\right) = 0. \tag{1.3}$$

Thus the scattering energy depends on the intensity parameter ν^2, the laser frequency ω, the integer r, the energy of the incoming electrons and the angles ψ and ϕ.

In this paper we shall examine how the scattering energies depend on the various laser and electron parameters and in which parameter regions the changes of the scattering energy compared to free Møller scattering are large enough for experimental verification. The special case $r = 0$ is treated in § 2. Results for the general case are presented in § 3.

These intensity-dependent changes of the scattering energy are not merely kinematical effects as one might suppose; the Møller cross section in a laser field is an incoherent sum of partial cross sections corresponding to the r-dependent scattering energies: $d\sigma = \Sigma_r \, d\sigma_r$. The magnitude of $d\sigma_r$ gives the probability with which an electron is scattered with energy $E_1'(r)$. But the knowledge of the largest r for which $d\sigma_r$ is still of relevant magnitude is crucial for the determination of the largest change in scattering energy. Hence there is close correlation between the convergence of the series of $d\sigma_r$, i.e. how many r contribute to the sum, and the maximum deviation of E_1' from the corresponding energy in free Møller scattering.

Due to technical difficulties the differential cross sections $d\sigma_r/d\Omega$ for high laser intensities could only be calculated for non-relativistic energies, so that the convergence of the series is only known for this energy region. Therefore all considerations below are restricted to non-relativistic electrons.

2. The scattering energies for $r = 0$

In this special case the electrons emit as many laser photons as they absorb while traversing the laser field. Thus only the collective effect of the intensity-dependent shift of the electron energies can be examined. If equation (1.3) is cast in the form of a polynomial, one obtains an equation with fourth powers in E_1' and very complicated coefficients. Therefore a non-relativistic approximation formula will be given and in addition numerical results gained by iteration programs. In both cases it is easier to calculate first the momenta of the scattered electrons and then the energies. Appropriate (dimensionless) parameters for these calculations are

$$p := |\boldsymbol{p}|/m, \qquad p_1' := |\boldsymbol{p}_1'|/m.$$

(If c is not set equal to one, p is for small velocities approximately equal to v/c.) The non-relativistic expansion of (1.3) yields

$$p' = \left(\frac{1+\nu^2\cos^2\psi}{1+\nu^2\cos^2\phi}\right)^{1/2} p + \nu^2 \cos\phi \frac{(1+\nu^2)(\sin^2\phi - \sin^2\psi)}{2(1+\nu^2\cos^2\phi)^2} p^2 + O(p^3). \tag{2.1}$$

We shall discuss the relative difference between the kinetic energies of scattered and incoming particles:

$$Z(r) := (E'_{1\,\mathrm{kin}}(r) - E_{\mathrm{kin}})/E_{\mathrm{kin}}.$$

With (2.1) we obtain for $Z_0 := Z(r=0)$

$$Z_0 = \nu^2 \frac{\cos^2\psi - \cos^2\phi}{1+\nu^2\cos^2\phi} + \nu^2\cos\phi\left(\frac{1+\nu^2\cos^2\psi}{1+\nu^2\cos^2\phi}\right)^{1/2} \frac{(1+\nu^2)(\sin^2\phi - \sin^2\psi)}{(1+\nu^2\cos^2\phi)^2} + O(p^2). \tag{2.2}$$

The maximum of the positive values of Z is found for $\psi = 0°$, $\phi = 90°$, i.e. if the two electron beams are shot parallel and antiparallel to the laser axis into the field and are observed at right angles to the axis:

$$Z_{0\,\max} = \nu^2 + O(p^2).$$

Hence for $\nu^2 = 1$ the kinetic energy of a scattered electron differs from the initial value by 100% (whereas these energies coincide in free Møller scattering). For lasers with an intensity below $\nu^2 = 10^{-2}$ a measurement of this energy shift will not be possible.

The minimum of Z is obtained for $\psi = 90°$, $\phi = 0°$:

$$Z_{0\ \min} = -\frac{\nu^2}{1+\nu^2} - p\frac{\nu^2}{(1+\nu^2)^{3/2}} + O(p^2).$$

Here the kinetic energy of one scattered electron is about 50% lower for $\nu^2 = 1$ than $E_{\rm kin}$.

In figure 1 the numerical results for Z_0 as a function of ϕ are presented for $p = 4{\cdot}47 \times 10^{-2}$, corresponding to a kinetic energy of 511 eV and for three angles ψ.

The graphs of figure 1 are nearly independent of p. Below $p \approx 0{\cdot}1$ the p term of (2.2) does not disturb the symmetry of the ϕ curves.

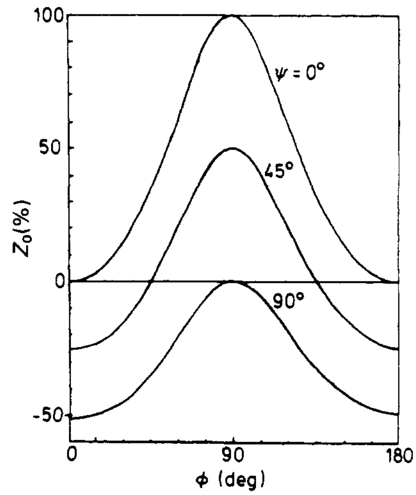

Figure 1. Relative change of kinetic energy Z_0 as a function of ϕ for $\nu^2 = 1$, $E_{\rm kin} = 511$ eV and three angles ψ.

If the energy and momentum of one scattered electron are known, the momentum of the second electron is easily calculated. From (1.1) we have

$$\boldsymbol{p}'_2 = -\boldsymbol{p}'_1 + \boldsymbol{n}(E'_2 + E'_1 - 2E) \tag{2.3}$$

where $\boldsymbol{n}$ is a unit vector in the direction of the laser beam. For $p'_2 := |\boldsymbol{p}'_2|/m$ we obtain $p'_2 = p'_1 + (p^2 - p'^2_1)\cos\phi + O(p^3)$. Since the magnitude of p'_2 differs from p'_1 only by terms proportional to p^2 and since the change in direction is small according to (2.3), we obtain

$$\boldsymbol{p}'_2 \approx -\boldsymbol{p}'_1$$

similar to free Møller scattering.

3. The scattering energy for $r \neq 0$

For $r \neq 0$ a larger energy change can be expected, since in addition to the 'mass shift' of the electrons in the laser field each electron can absorb or emit laser quanta. According to (1.1) $r > 0$ means that r laser photons are absorbed by the electrons, whereas in the case of $r<0$, r laser photons are emitted. As the energy of the laser quanta is small compared to the electron rest mass, the kinetic energy of the electrons will be affected mainly in the non-relativistic energy region by the absorption or emission of laser quanta. The change in energy will be large for high values of r and for electron kinetic energies of the same order of magnitude as the photon energy. Therefore these angles ψ, ϕ and θ will be interesting at those values for which the convergence of $d\sigma_r$ is weak, i.e. where large r contributes to the cross section. For a Nd-glass laser, e.g. with $\omega = 1{\cdot}9\times10^{15}\ \mathrm{s}^{-1}$ and for electrons with a kinetic energy of 5 eV, $r=10$ is sufficient for $r\omega = E_{\mathrm{kin}}$, whereas for 0·5 keV electrons, $r=1000$ is needed.

Before presenting approximation formulae and numerical results for the scattering energy, the experimental consequences of the correlation between energy shift and ν convergence will be discussed. If $d\sigma_r$ is as large for several values of r as it is for $r=0$, one finds—for fixed laser and electron parameters—electrons with different energies $E'(r)$ in the same solid angle. For non-relativistic electrons this discrete distribution of energies over a certain energy band should be broader than the statistical error of the energies detected experimentally.

4. Non-relativistic approximation formulae

Expanding (1.3) to second order in p and p'_1 and neglecting quadratic terms in $\delta r := r(\omega/m)$, we obtain

$$p'_1 = \frac{\delta r\cos\phi}{2(1+\nu^2\cos^2\phi)} + \left(\frac{p^2(1+\nu^2\cos^2\psi)+\delta r[1+p^2(1+\sin^2\psi)]}{1+\nu^2\cos^2\phi}\right)^{1/2} + \mathrm{O}(p^2, \delta r^2). \tag{4.1}$$

In order to expand also the square root expression the cases $|\delta r/p^2|>1$ and $|\delta r/p^2|<1$ must be treated separately. The first case means that the energy of the net number of photons emitted or absorbed by the two electrons is larger than the kinetic energy of the electrons. Hence large deviations from Z_0 may be expected, whereas for $|\delta r/p^2|<1$ the additional changes in energy will be smaller.

For $|\delta r/p^2|>1$ we have

$$p'_1 = \frac{\delta r\cos\phi}{2(1+\nu^2\cos^2\phi)} - \left(\frac{\delta r}{1+\nu^2\cos^2\phi}\right)^{1/2} + (\delta r)^{1/2}\frac{p^2(1+\nu^2\cos^2\psi)}{2\delta r(1+\nu^2\cos^2\phi)^{1/2}} + \mathrm{O}(p^2, \delta r^2, (p^2/\delta r)^2),$$

$$Z_r \approx Z_0 + \left(\frac{\delta r}{p^2}\right)\frac{1}{1+\nu^2\cos^2\phi} + \left(\frac{p^2}{\delta r}\right)\frac{(1+\nu^2\cos^2\psi)^2}{4(1+\nu^2\cos^2\phi)} + \frac{(\delta r)^{3/2}\cos\phi}{p^2(1+\nu^2\cos^2\phi)^{3/2}}. \tag{4.2}$$

The additional terms are mainly determined by the magnitude of $\delta r/p^2$. For example: for 5 eV electrons, $\omega/m = 10^{-6}$ (Nd–glass laser) and $r=100$ one obtains $\delta r/p^2 = 5$. Since in some cases such high values of r seem to be relevant because of the magnitude of the corresponding cross section, Z_r may be much larger than Z_0.

For $|\delta r/p^2|<1$ we have

$$p'_1 = p\left(\frac{1+\nu^2\cos^2\psi}{1+\nu^2\cos^2\phi}\right)^{1/2} + \frac{\delta r\cos\phi}{2(1+\nu^2\cos^2\phi)} + \left(\frac{\delta r}{p}\right)\frac{[1+p^2(1+\sin^2\psi)]}{2(1+\nu^2\cos^2\psi)^{1/2}} + \mathrm{O}(p^2, \delta r^2, (\delta r/p^2)^2),$$

$$Z_r \approx Z_0 + \left(\frac{\delta r}{p^2}\right)\frac{[1+p^2(1+\sin^2\psi)]}{(1+\nu^2\cos^2\phi)^{1/2}} + \left(\frac{\delta r}{p^2}\right)^2\frac{[1+p^2(1+\sin^2\psi)]^2}{4(1+\nu^2\cos^2\phi)}. \tag{4.3}$$

Again the corrections for Z_0 are determined mainly by $\delta r/p^2$.

For the measurement of the discrete energy spectrum $E'(r)$ the spacing of the energies is interesting. The difference between two adjacent energy values is given by

$$\Delta E'_{\mathrm{kin}}(r) \approx (\omega/2m)(1+\nu^2\cos^2\phi)^{-\tau}$$

with

$$\tau = \begin{cases} 1 & \text{for } |\delta r/p^2| > 1 \\ \frac{1}{2} & \text{for } |\delta r/p^2| < 1. \end{cases}$$

Thus the magnitude of $\Delta E'_{\mathrm{kin}}$ is determined by the laser frequency. For an infrared laser with $\omega/m = 10^{-6}$, $\omega/2m$ corresponds to an energy difference of 0·25 eV, whereas for an ultraviolet laser with $\omega/m \simeq 10^{-5}$ one finds 2·5 eV. As in the case of the resonances the ultraviolet frequencies appear to be more favourable for experiment.

Finally we wish to mention that a lower limit for r exists, since the two electrons can emit at most a number of laser photons corresponding to their kinetic energy. This lower limit follows from (4.1). One obtains

$$r = -(mp^2/\omega)(1+\nu^2\cos^2\psi) + \mathrm{O}(p^4).$$

For example, the lower limit for $\omega/m = 10^{-5}$, $E_{\mathrm{kin}} = 25{\cdot}5$ eV and $\psi = 90°$ is $r = -10$. In many cases, however, the series of the partial cross sections converges before reaching this limit.

5. Numerical results

Figure 2 gives $Z(r)$ as a function of ϕ. Since for each angle ϕ the set of values r for which the partial cross sections $\mathrm{d}\sigma_r$ must be summed is completely different, one should show a separate diagram of how Z depends on r for each angle ϕ. Here another way was chosen: a representation as in figure 1, but with two values Z for each angle ϕ. For the positive values of r the number $r_{\max}$ is determined for which $\mathrm{d}\sigma_{r\ \max}$ is still of a relevant magnitude, but where the further $\mathrm{d}\sigma_r$ with $r > r_{\max}$ can be neglected; likewise $r_{\min}$ for the negative values of r. One may proceed like that because of the manner of convergence of $\mathrm{d}\sigma_r$: up to a certain value of r all $\mathrm{d}\sigma_r$ are of the same order of magnitude as $\mathrm{d}\sigma_0$. They differ by a factor of less than ten. The following $\mathrm{d}\sigma_r$ however decrease very rapidly to zero. There is, of course, some freedom in the definition of $r_{\max}$ and $r_{\min}$. We choose $\mathrm{d}\sigma_{r\ \max} \approx \frac{1}{10}\max(\mathrm{d}\sigma_r)$, and analogously $r_{\min}$. Although with $\nu^2 = 10^{-2}$ a relatively small laser intensity was chosen, the energy change amounts to 20% at the maximum ($\phi = 135°$). For this angle Z_0 is zero so that the energy difference is caused only by the absorption of laser quanta.

According to the definition above there is a number of further points $Z(r)$ for each angle ϕ between the graphs for $r_{\max}$ and $r_{\min}$. In experiments one should find all kinetic energies $E'_{\mathrm{kin}}(r)$ for $r \in [r_{\min}, r_{\max}]$. The probability with which each energy occurs is given by $\mathrm{d}\sigma_r/\mathrm{d}\Omega$. For the values of r between $r_{\max}$ and $r_{\min}$ the magnitude of $\mathrm{d}\sigma_r/\mathrm{d}\Omega$ differs only by a factor ten by definition. There will of course be further energies beyond $r_{\max}$ and $r_{\min}$ but they will occur with a much smaller probability because of the rapid decrease of $\mathrm{d}\sigma_r$. To give an impression of the magnitude of $r_{\max}$ and $r_{\min}$ their values from figure 2 are given in table 1.

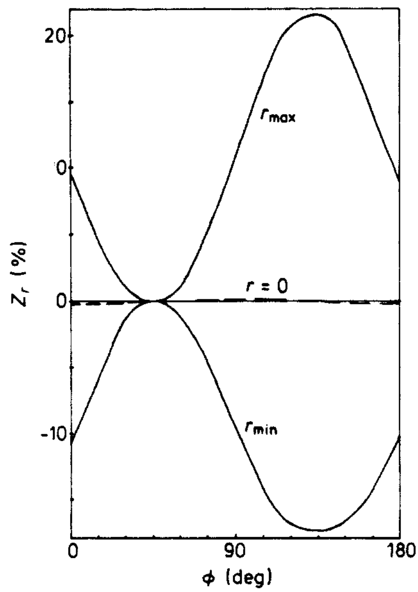

Figure 2. $Z(r)$ as a function of ϕ for $\nu^2 = 0{\cdot}01$, $\omega/m = 7{\cdot}8 \times 10^{14}\,\mathrm{s}^{-1}$, $E_{kin} = 511$ eV, $\psi = 45°$, $\theta = |\psi - \phi|$.

Table 1. Maximum and minimum values of r as a function of scattering angle ϕ.

ϕ	r_{max}	r_{min}
0°	200	−200
22·5°	60	−60
45°	0	0
67·5°	50	−60
90°	200	−200
112·5°	380	−330
135°	450	−360
157·5°	370	−320
180°	200	−200

Calculations were also done for the same parameters as in figure 2, but for an ultraviolet laser with $\omega = 8{\cdot}0 \times 10^{15}\,\mathrm{s}^{-1}$ ($(\omega/m) = 10^{-5}$). The values of r_{max} and r_{min} turned out to be smaller by a factor ten so that the products $r_{max}(\omega/m)$ and $r_{min}(\omega/m)$ which mainly determine Z_r remained the same as for $(\omega/m) = 10^{-6}$. Thus nearly exactly the same graphs were obtained for the ultraviolet laser as in figure 2.

For similar reasons Z_r remains approximately the same when the electron energy is changed (within the non-relativistic region). Again one obtains the graphs of figures 2 for $\nu^2 = 10^{-2}$ and 5 eV electrons because the cross sections converge more rapidly for smaller electron energies. For changes in ω the product $r(\omega/m)$ remained constant. Here it is r_{max}/p^2 so that again the decisive factor $r\omega/mp^2$ does not change (cf (3.2) and (3.3)).

For technical computing reasons it is easier to calculate the convergence of the cross sections for $\nu^2 = 1$ with high laser frequencies and low electron energies. From these results one can draw inferences for higher electron energies and lower frequencies according to the arguments above. A plot is given in figure 3. Here the cross sections converge very rapidly because of the ultraviolet frequency and the small electron energy of 5 eV: r_{max} does not exceed $r = 10$ and r_{min} is limited to values above -3 (the lower limit causes the asymmetric form of the r_{min} graph relative to the graph for r_{max}).

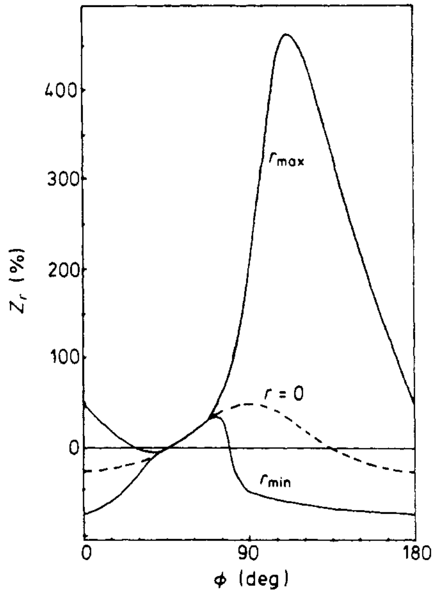

Figure 3. $Z(r)$ as a function of ϕ for $\nu^2 = 1$, $\omega/m = 8\times10^{15}\ s^{-1}$, $E_{kin} = 5$ eV, $\psi = 45°$, $\theta = |\psi - \phi|$.

The energy change is quite large for $\nu^2 = 1$: the maximum of Z_r amounts to 460% and is much larger than the maximum of Z_0. Nevertheless the ν^2 shift of the electron energies is an essential contribution here. So, for instance, the maximum of Z_r is shifted from $\phi = 135°$ towards 95° which corresponds to the maximum of Z_0. Part of the r_{max} and r_{min} graphs coincides with the $r = 0$ graph; from $\phi = 45°$ to 70° r_{max} and r_{min} are zero. In this region the energy change is caused exclusively by the ν^2 shift. This is an important point. For all other angles—and also for the $\nu^2 = 10^{-2}$ graphs of figure 2—the ν^2 shift is covered by the discrete energy spectrum for $r \neq 0$. Therefore a direct experimental test for the ν^2 shift of the scattering energy is probably only possible for the interval of ϕ where the three graphs coincide. This ϕ interval gets narrower, however, for higher energies and lower frequencies. In addition Z_0 also decreases to zero.

But in any case it should be possible to verify experimentally the considerable energy change caused by the ν^2 shift and the absorption or emission of laser quanta together—and not only for $\nu^2 = 1$, but already for $\nu^2 = 10^{-2}$ as figure 2 shows.

Finally, a problematical point must be mentioned. At the root of all calculations is the assumption of an infinitely extended laser field. But in reality the electrons spend only a finite time within the laser field. Therefore the maximal number of laser photons that can be absorbed or emitted by the electrons in this short time interval may be smaller than predicted from the pure mathematical considerations concerning the convergence of the differential cross sections. If that is the case, the maximum of Z_r will not be as high as in figures 2 and 3, but the magnitude of the pure ν^2 shift of the scattering energy might be more important that is implied by the graphs of figures 2 and 3.

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